Equivalent granular state parameter and undrained responses for sand with fines

Équivalent état granulaire paramètre undrained et réponses pour le sable avec des amendes

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ABSTRACT

Recent publication reported that void ratio is not a good state variable for sand with fines. Thus, equivalent granular void ratio, e^* has been proposed to resolve this problem. The Steady State data points in the e^*-log(p') space can be described by a single trend irrespective of fines content, f_c, provided $f_c \leq$ TFC. This single trend line is referred to equivalent granular steady state line, EG-SSL. However, whether the EG-SSL can be used within the context of critical state soil mechanics (CSSM) for predicting undrained behaviour has to be investigated. An equivalent granular state parameter, ψ^*, defined using EG-SSL as the reference is proposed in this paper. It is found that ψ^* can successfully predict undrained responses of sand with fines in undrained shearing.

RÉSUMÉ

Publication récente a indiqué que nul ratio n'est pas une bonne variable d'état pour le sable avec des amendes. Ainsi, l'équivalent granular void ratio, e^* a été proposée pour résoudre ce problème. L'état d'équilibre des points de données dans le e^*-log(p') espace peut être décrite par une tendance unique, indépendamment du montant des amendes de contenu, f_c, à condition $f_c \leq$ TFC. Cette simple ligne de tendance est visée à l'état d'équilibre équivalent granulaire ligne, EG-SSL. Toutefois, si le EG-SSL peut être utilisé dans le contexte de la situation critique mécanique des sols (CSSM) undrained pour prédire le comportement doit être étudiée. Un équivalent état granulaire paramètre, ψ^*, définis à l'aide d'EG-SSL de référence est proposé dans le présent document. Il est constaté que ψ^* peut prédire avec succès undrained réponses de sable avec des amendes en undrained cisaillement.

Keywords : sand, fines, equivalent granular, steady state, state parameter

1 INTRODUCTION

Traditionally, void ratio, e has been used as a state variable for predicting the static liquefaction behaviour, also refers to as instability behaviour, of sand using the Critical State (Steady State) framework. However, recent publications unambiguously showed that the steady state line (SSL) in the e-log(p') space is dependent on fines content, where p' is the effective mean stress. At the lower end of fines content, f_c, the SSL moved downward with increase in f_c, but the direction of movement eventually reversed. The fines content at this reversal point is termed as threshold fines content, TFC. Thus, a family of SSLs may be needed for predicting the static liquefaction behaviour for sand with fines. Many researchers proposed to use the equivalent granular void ratio, e^* as an alternative to e in an attempt to obtain a single trend line for SS data points in the e^*-log(p') space, irrespective of f_c, provided $f_c \leq$ TFC (Thevanayagam et al. 2002; Rahman and Lo 2007a; Rahman and Lo 2007b; Rahman and Lo 2008a). The definition and determination of e^* will be discussed in the next section.

This single trend line for SS data points for sand with a range of fines contents was referred to as the equivalent granular steady state line, EG-SSL, by Rahman and Lo (2008b). However, having a single trend line in the for the SS data points e^*-log(p') space does not guarantee that the EG-SSL can be used in the context of the critical state soil mechanics (CSSM) framework in predicting the behaviour of sand with fines. The objective of this paper is to examine this issue. First, the EG-SSL is used to predict undrained behaviour (flow, limited flow and non-flow) of sand with fines. Then, the definition of equivalent granular state parameter, ψ^*, using EG-SSL as the reference was proposed. The correlation between ψ^* and instability stress ratio, η_{IS}, in undrained shearing were examined and a good correlation observed between them.

2 LITERATURE REVIEW

The predecessor of e^* is intergranular void ratio, e_g, defined by Thevanayagam (1998) as:

$$e_g = \frac{e + f_c}{1 - f_c} \tag{1}$$

where, e is void ratio, f_c fines content. Equation (1) is based on assumption that the fines can be approximated as void spaces, i.e. their contribution to the force structure can be neglected. The intergranular void ratio, e_g was also being referred to as void ratio of granular phase (Mitchell 1976), skeleton void ratio (Kuerbis et al. 1988), granular void ratio (Georgiannou et al. 1990). The work of Pitman (1994), among others, showed that e_g can function as an alternative to e provided that f_c content is low relative to the void space formed by the hoist sand.

At higher fines content (relative to the void space), the fines begin to participate in the force structure. Therefore, Thevanayagam et al. (2002) proposed the use of equivalent granular state parameter, e^* defined by Equation (2) below as a better alternative to e:

$$e^* = \frac{e + (1-b) f_c}{1 - (1-b) f_c} \tag{2}$$

where, b represents the fraction of fines that are active in the force structure of the sand-fines mixture. e^* was also being referred to as, equivalent inter-granular contact void ratio (Thevanayagam et al. 2002), corrected intergranular void ratio

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(Yang et al. 2006), equivalent inter-granular contact index void ratio (Thevanayagam 2007), equivalent granular void ratio (Rahman and Lo 2008b; Rahman et al. 2008a; Rahman et al. 2008b). The basic assumption of e^* as defined in Equation (2) requires $1 \geq b \geq 0$, and that $f_c <$ TFC.

By setting $b = 0$, the definition of e^* reduced to that of e_g. This implies, $b = 0$ (which yield $e^* = e_g$) is an adequate approximation at low fines content, but the non-zero value of b should be considered at higher fines content. A corollary is that b is a function of f_c, and this is also consistent with the 'fines-in-sand' soil fabric. However, most of the b-values reported in various publications were not correlated to f_c. Furthermore, they were deduced by back-analysis of extensive triaxial test sets covering a range of fines content. Therefore, such a b value is in fact an average value for the range of fines contents covered in a particular publication and was selected to achieve a desired relationship for a range of f_c. This approach sometimes led to unreasonable b values such as a very high b-value of 0.7 being achieved for a low f_c of 9% and a negative b-value of -0.8 (Ni et al. 2004).

Rahman et al. (2008a), based on a re-analysis of the experimental data of McGeary (1961) on binary packing studies, concluded that b is a function of f_c and particle diameter ratio, χ, i.e. $b = F(\chi, f_c)$. Furthermore, the functional relationship has to possess certain mathematical attributes. A semi-empirical equation expressed as Equation (3) was proposed.

$$b = \left[1 - \exp\left(-\mu \frac{(f_c / TFC)^{n_b}}{k}\right)\right]\left(r \frac{f_c}{TFC}\right)^r \quad (3)$$

where, $k = (1 - r^{0.25})$, TFC = threshold fines content and r = diameter ratio, $1/\chi$.

To further simplifies the equation, one can assign $n_b = 1$ and have μ as the calibration constant. Rahman and Lo (2008b) showed that a single value of $\mu = 0.30$ could be used for a large number of data sets published in various literatures. Thus, the Equation (3) can be simplified to:

$$b = \left[1 - \exp\left(-0.3 \frac{(f_c / TFC)}{k}\right)\right]\left(r \frac{f_c}{TFC}\right)^r \quad (3a)$$

The prediction of TFC is another issue to be addressed before Equation (3a) can be used. Rahman and Lo (2008b) suggested the use of the following Equation based on calibration with ten data sets.

$$TFC = A_{TFC}\left(\frac{1}{1 + e^{\alpha - \beta\chi}} + \frac{1}{\chi}\right) \quad (4)$$

The coefficient of A_{TFC} can be determined from the asymptote of the plot and this gives, $A_{TFC} = 0.40$. The other two parameters α and β are determined by curve fitting and this gave $\alpha = 0.50$ and $\beta = 0.13$.

Rahman and Lo (2008b) verified the concept of a single EG-SSL by examining ten data sets from seven different countries around the world. Equation (3a) and (4) was used to calculate the b-value which in turn enables the conversion of e to e^*. The objective of this paper is i) to further verify the validity of a single EG-SSL for a sand-fines mixture in order to address the limitations of the published data sets and ii) examine whether the EG-SSL can be used in the context of CSSM framework in predicting instability behaviour in undrained loading. Along this line, an equivalent granular state parameter, ψ^*, is introduced as an alternative to the state parameter ψ as proposed by Been and Jefferies (1985).

3 EXPERIMENTAL STUDY

A series of experiments have been done on sand with a range of fines content: 0%, 15%, 20%, and 30%.

3.1 Tested Material

The host sand was Sydney sand, a medium quartz sand is a clean sand (SP) and it's index properties can be found in Lo et al. (1989). The fines is a specially designed low plasticity fines (PI=27, LL=54) with a uniformity coefficient 12.56. It is composed of 2/3 of well-graded silt from the Majura River and 1/3 commercial kaolin. Note that the published data sets examined in Rahman and Lo (2008b) largely had non-plastic fines with small uniformity coefficient. Thus, this sand-fines mixture addressed the limitations of the published data sets. Furthermore, Equation (4) gives TFC = 0.40 and this enable a wide range of fines content (0% to 30%) can be covered in this experimental investigation.

3.2 Experimental Procedure

A strain controlled triaxial loading system with fully automated data logging facilities was used for this study. Axial load was measured with an internal load cell. The axial deformation was measured by two independent means: a pair of internal LVDTs mounted directly across the platens and an external LVDT. The former was used in the early stage of shearing whereas the latter was used at large deformation. Cell pressure was controlled by a large capacity Digital Pressure Volume Controllers (DPVC). The pore pressure line was connected to a small capacity DPVC for controlling back pressure (and measuring the volume change) at the consolidation stage and for imposing an undrained condition and measuring the resultant pore pressure response during shearing. Two pressure transducers were also used to verify pore pressure equilibrium.

A modified moist tamping method was used for specimen preparation to ensure uniformity of the specimen. To accurately control the void ratio and, a total of 10 layers of predetermined quantities of moist soil were worked into a prescribed thickness. Details of specimen preparation method can be found at Rahman et al. (2008a). Enlarged end platens with free ends, as describe by Lo et al. (1989), was used to minimize end restraint. Liquid rubber technique was used to minimize bedding and membrane penetration error, and also to ensure even seating at the top platen.

Saturation of the specimen was accomplished in two stages. Stage-I consisted of carbon dioxide percolation for at least 20 minutes followed by vacuum flushing with a low head. Stage-2 was back pressure application to achieve a Skempton B-value of at least 0.98. Special efforts were made to measure the void ratio accurately, which is an essential requirement for this study.

4 EQUIVALENT GRANULAR STEADY STATE LINE, EG-SSL

The SS data points in the e^*-log(p') space are shown in Figure 1. Equation (2), (3a) and (4) were used to calculate e^*. Evidently, all SS data points in e^*-log(p') space followed a single trend i.e. a single EG-SSL was obtained. However, this EG-SSL is not a straight line, but rather a curve which is consistent with many publications (Wang et al. 2002; Bobei and Lo 2005). Thus, the curvature should be born in mind though it is usually referred to as line. The EG-SSL can be presented by the following Equation

$$e^* = 0.908 - 0.0266\left(\frac{p'}{p_a}\right)^{0.7} \quad (5)$$

where, p' = mean effective stress, p_a = atmospheric pressure.

4.1 Prediction of Undrained Behaviour

In order to evaluate EG-SSL for sand with a wide range of $f_c \leq$ TFC, the initial state (i.e. just prior to shearing) of each tests was plotted as a data point in Figure 2. The EG-SSL in e^*-

log(p') space were also plotted in this figure. The data points for tests with flow behaviour were all located well above the EG-SSL. Tests with non-flow were represented by data points located well below the EG-SSL. The data points for tests with limited-flow behaviour were plotted around the EG-SSL. This demonstrated that the EG-SSL, in conjunction with the initial states, can be used within the CSSM framework in predicting the overall undrained behaviour patterns.

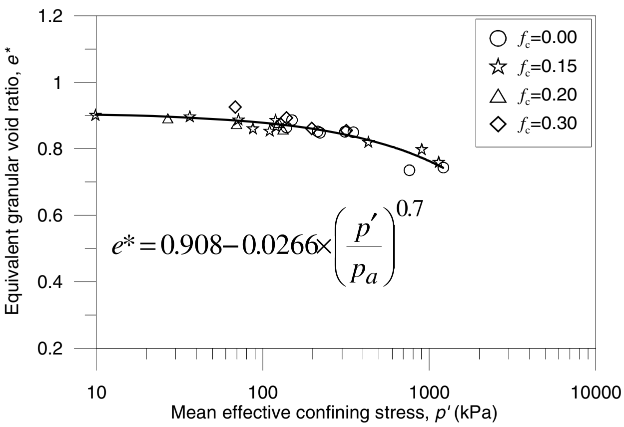

Figure 1. The EG-SSL for sand with fines in e^*-log(p') space.

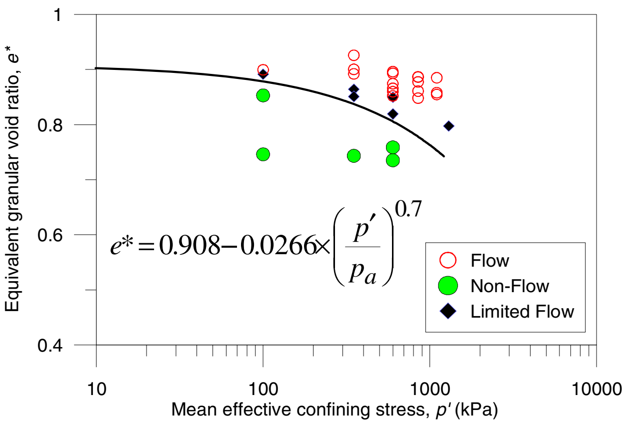

Figure 2. Initial state of all tests for sand and sand with fines (up to 30%) relative to EG-SSL.

4.2 *Equivalent Granular State Parameter, ψ^**

This paper hypothesize that EG-SSL can be used as the single "alternative SSL" in the e^*-log(p') space in the context of CSSM framework. Thus along the line of Been and Jefferies (1985), we introduced the concept of equivalent granular state parameter, ψ^*, defined as:

$$\psi^* = e^* - e^*_{SS} \tag{6}$$

where, $e^*_{SS} = e^*$ on the EG-SSL at same p'. The definition of ψ^* is schematically illustrated in Figure 3. We can then denoted the initial value of ψ^* by $\psi^*(0)$. It is pertinent to note that having the initial state plotted above the EG-SSL is equivalent to having $\psi^*(0) > 0$, and that $\psi^*(0) < 0$ means having the initial state located below the EG-SSL. Thus the definition of Equation (6) is consistent with the findings of the previous verification.

4.3 *Relation between $\psi^*(0)$ and Instability stress ratio, η_{IS}*

In undrained shearing of a specimen manifesting flow or limited flow behaviour, the point corresponding to peak deviator stress is also the onset of instability, sometimes referred to static liquefaction. For a given clean sand being sheared at a given initial void ratio, the effective stress ratio, $\eta = q/p'$, at onset of instability is an approximately constant value. This effective stress ratio is referred to as the instability stress ratio denoted as η_{IS}. To further verify the concept of ψ^*, the relation between $\psi^*(0)$ and instability stress ratio, η_{IS}, is examined by plotting all the (η_{IS}, $\psi^*(0)$) data points as presented in Figure 4. Both flow and limited flow data points are included in the plot. A single relationship irrespective of f_c is obtained. Although there is some scatter around the trend line, the scatter does not show any correlation with f_c. Furthermore, η_{IS} decreases with increase in $\psi^*(0)$ which is consistent with expectation and our hypothesis.

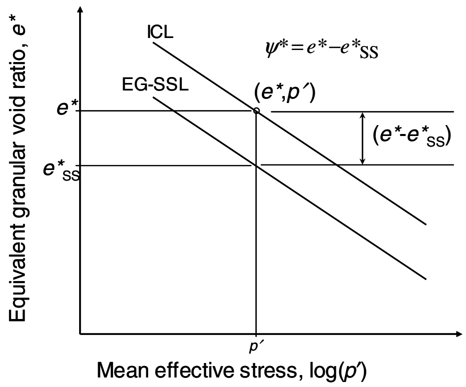

Figure 3. Definition of equivalent granular state parameter, ψ^*

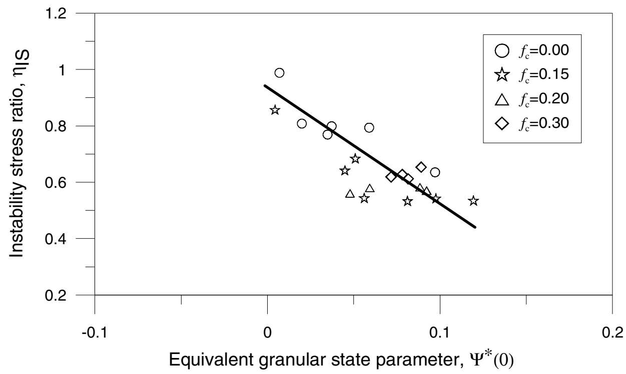

Figure 4. The relation between equivalent granular state parameter, $\psi^*(0)$ and instability stress ratio, η_{IS} for sand with fines, $f_c \leq$ TFC.

5 CONCLUSIONS

The challenges associated with predicting the instability behaviour of sand with fines under the CSSM framework are examined. An experimental investigation has been done on a sand with low plasticity fines with fines content ranging from 0% to 30%. The findings are as follows.

- The prediction equation of b can be represented as $b = F(\chi, f_c)$, and the conversion from e to e^* can be done without the need of substantial data sets for back-analysis.
- The concept of equivalent granular steady state line, EQ-SSL, which is a single relationship, for describing SS data points in the e^*-log(p') space is confirmed.
- The concept and definition of equivalent granular state parameter, ψ^*, for sand with fines was proposed. Preliminary test results showed that the value of ψ^* at the start of shearing can be used as a predictor of undrained behaviour irrespective of fines contents and in the content of CSSM.

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